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IONOSPHERE PLASMA DENSITY ESTIMATION BY RAY TRACING OPTIMIZATION

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Abstract

In recent years, several studies have tried to estimate volumetric electron density by methods of refraction tomography on an HF network. These methods involve a dynamic optimization problem where the ray tracing equations have to be solved in every optimization step [1].

Furthermore, to improve the estimates, data from incoherent scatter radars and GPS can also be assimilated. However, the computational complexity involved in these estimates is considerable. Even though some efforts have been implemented to reduce this complexity, it is clear that new methods have to be explored.

This work focuses our efforts on the inverse process. Instead of using sensitivity analysis, we propose a direct collocation approach, where the points on the transmitter and receiver can be fixed, therefore, eliminating the chances of the extreme misfire.

Introduction

Ray tracing can be performed using direct variational methods and numerical relaxation, e.g. Coleman (2011), or converting the variational problem into a system of coupled first-order differential equations using the principles of analytic mechanics, e.g. Landau & Lifshitz (1976).

On the other hand, Hysell [1] developed a robust optimization model that is the direct variational sensitivity analysis for ray tracing. This model uses real-time empirical data and sensitivity analysis with the signal power to fit rays. However, it is computationally expensive.

In this work, it is proposed to develop a new optimization model based on the collocation method. And use the simplified variational principle (unmagnetized collisionless) developed by Coleman as a comparison metric.

Method

In the work developed by Coleman in 2011 [2], a powerful ray-tracing technique is obtained by knowing the start and end points. This approach allows solving the system directly starting from the variational principle from which the Haselgrove equations are derived. The parameterized expression is found in eq.1.

$$\partial \int_A^B \mu_{(\bar{r},Y_p)} p_t \, \mathrm{d}s = 0 \quad \text{...(eq.1)}$$

Method

In eq.1, the variational principle is shown in terms of the geometry of the ray path only. The values of p_t and Y_p are reduced to solve the equation system with eq.2 and eq.3.

$$Y_t = p_t \left(Y_p - \frac{1}{2}\left(Y^2 - Y_p^2\right) \frac{d \ln(\mu^2)}{dY_p} \right) \quad \text{...(eq.2)}$$

$$1 = p_t \left(p_t - \frac{1}{2}\left(Y_t - Y_p p_t\right) \frac{d \ln(\mu^2)}{dY_p} \right) \quad \text{...(eq.3)}$$

Coleman's method reduces to knowing the following parameters: $X,\ Y,\ \bar{r}, \bar{t}, Y_p, p_t$

Thus, by defining an ionospheric model and knowing the aforementioned parameters, the functional evaluation is carried out in order to obtain the rays between the transmitter and the desired arrival point [fig. 2].

On the other hand, the development of optimal control for ray tracing with the collocation methods [3] are compared in fig. 1 with the shooting methods. In the collocation method the trajectory is approximated using a piecewise polynomial ($p_{(t)}$) fig. 2.

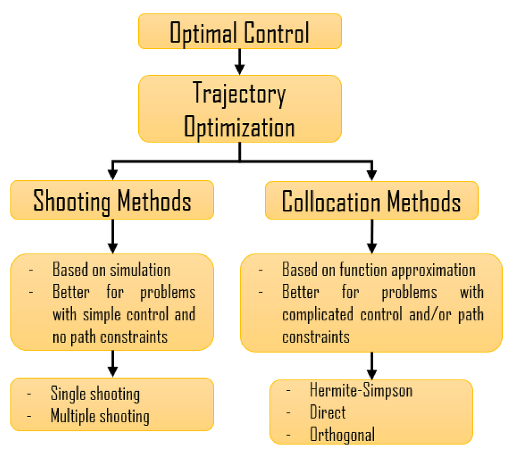

Figure 1: Methods for trajectory optimization

Physics is satisfied by requiring that the derivative of the state matches the derivative of the polynomial at each collocation point, the points that implicitly define the polynomial.

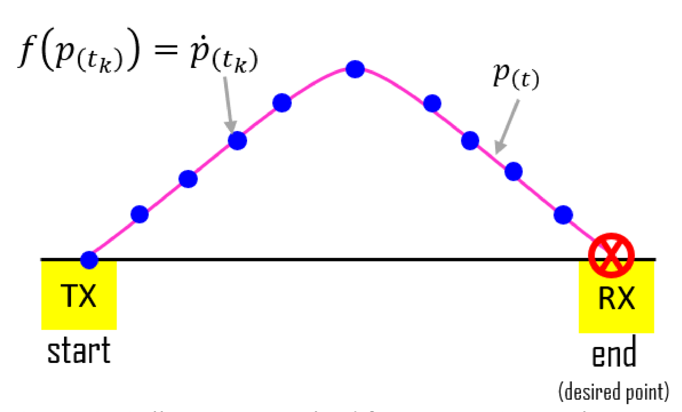

Figure 2: Collocation Method for ray tracing technique

Based on the work of Kelly (2017) [4], we show an example of how the calculation of optimal trajectory is obtained for a projectile trajectory. The first is the shooting method and the second is the collocation method proposed in this work.

The single shooting method [fig.3] is implemented using the standard 4th-order Runge-Kutta integration scheme for the simulation, and the total time is included as a decision variable.

The collocation method [fig.4] is implemented using the Hermite-Simpson quadrature. This is just one of many possible schemes.

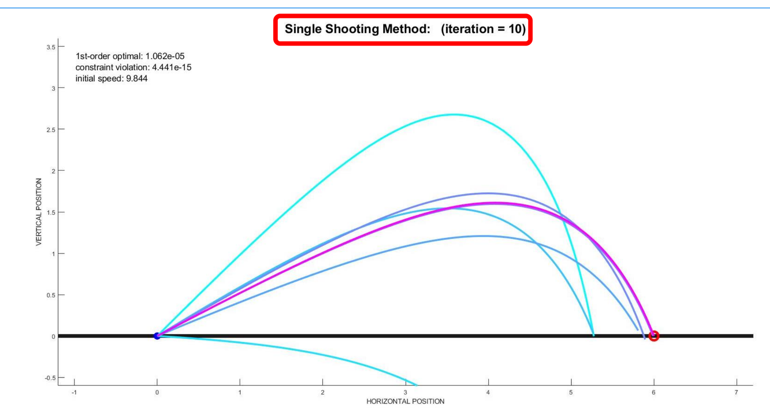

Figure 3: Single Shooting Method

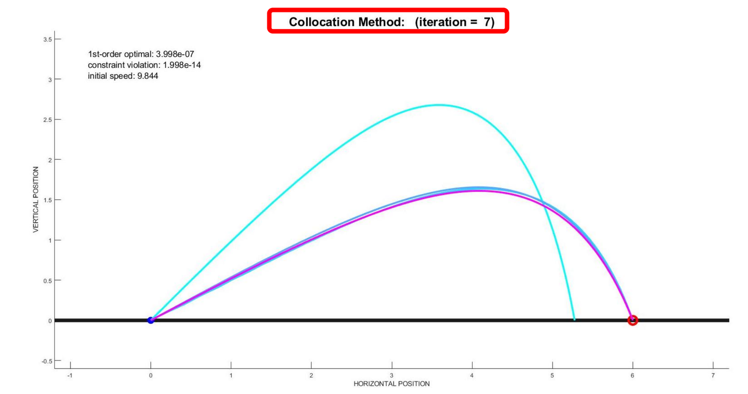

Figure 4: Collocation Method

From figures 3 and 4, in the upper red squares of each one, indicate the number of iterations. Being the collocation method of 7 iterations compared to the 10 iterations of the single shooting method to obtain the solution.

The scheme of work with which it is working is detailed below [fig. 5]. It consists mainly of testing the proposed model and comparing it with the simplified Coleman model.

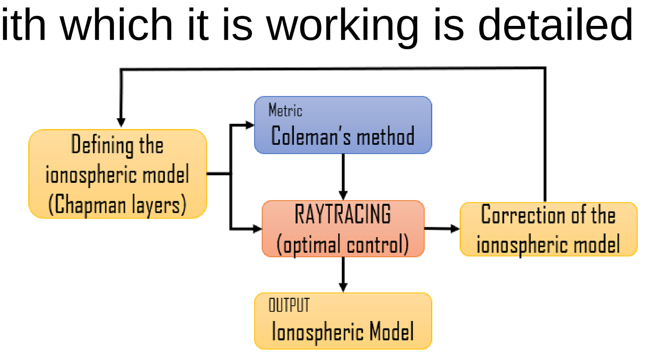

Figure 5: Proposed Methodology

Preliminary Results

The complexity of the modeling of the ionosphere is not crucial at this point of the work. For that reason, we are going to assume an unmagnetized collision less plasma on the ionosphere. However, to understand the dynamics between the ray tracing and the ionosphere layers, 2 layers were initially modeled: a lower layer, E at 145 km, while the upper layer, F at 300 km [fig. 6].

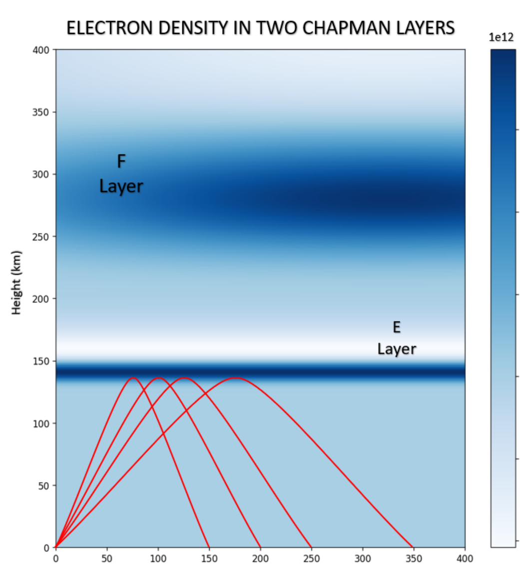

Figure 6: Ray Tracing, and E and F layer

A first approximation to the problem, we modeled using the single shotting method with conventional numerical methods such as Bisection and Secant.

Preliminary Results

To do this, we initially model the ionosphere with Chapman layers. A ray sweep was performed with 5° variations in elevation from 35° to 80° and keeping the azimuth angle constant at 30°. The operating frequency was 8 MHz (fig. 7).

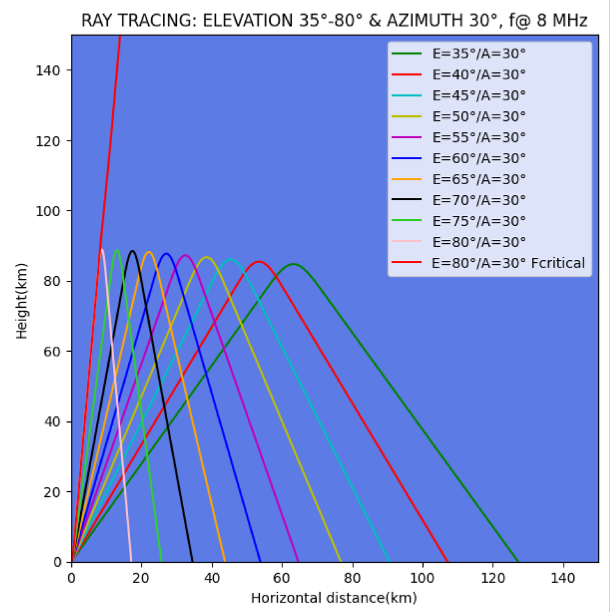

Figure 7: Ray tracing with variable elevation

The idea of the sweep is to choose the desired arrival point associated with an elevation angle. Then, we start from the inverse process by defining a function to determine the elevation angles of the emitted rays and thus correct the trajectory. Fig. 8 shows the results of one of the initial iterations.

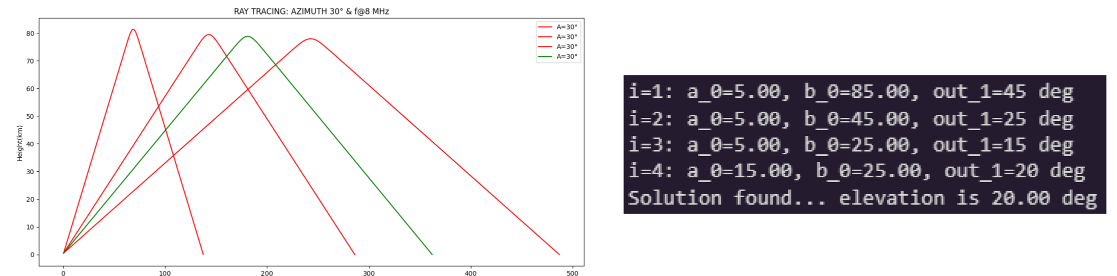

Figure 8: (a) Ray tracing function to determine elevation angle, (b) results

Summary

With the study of the state of the art of the collocation method and as a comparison metric, the simplified Coleman method, the methodology of fig. 5 has been worked with the preliminary results shown up to now.

We built a solver using collocation methods because it presents a new trajectory optimization over the shooting methods that are usually used for this type of problem. From Kelly's results (fig. 3, 4), the evaluation of both methods in ray tracing looks promising.

As future work, the integration of the collocation method is about to be completed for comparison with the simplified Coleman model. Also, it is planned to add other different ionospheric models, both Chapman and IRI or SAMI-3.

References

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